CAAM 335, Fall 2021, Homework 4

You may consult your CAAM 335 Course Notes and *Linear Algebra in Situ* by Steven Cox. MATLAB, Python, and other computational software are permitted on only some problems in this homework. You are not allowed to consult homework solutions from previous semesters.

If the homework contains programming assignments, turn in the code you wrote and the output it generates. Output must be well formatted. For example, figure axes must be appropriately scaled and must be labelled.

Submissions are due via Canvas on October 4, 2021.

Problem 1: (30 points)

(a) Compute, by hand, the LU-decomposition of

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & -1 & -3 \\ -3 & -2 & -9 \end{pmatrix}.$$

Show all steps of the computation. What are L and U?

(b) Suppose you have computed the LU-decompositon $A = LU$ of A. Describe how you can use it to solve $A^T x = f$.

(c) Let A be the matrix in part (a). Use your procedure in (b) to solve $A^T x = f$, where $f = (1,2,1)^T$.

Problem 2: (10+10 = 20 points)
An "unstable swing" is made up of the compressible bars labeled 1, 2, and 3 in the figures above. This swing is unstable, as we concluded in class with a similar example. In this problem, we make two attempts to stabilize the swing. (The gray regions denote rigid walls.)

(a) Add a vertical bar to arrive at the configuration shown in the left figure above. Compute the matrix A that relates displacements to elongations, and find all x's such that $Ax = 0$. Is this configuration stable?

(b) Instead, add a horizontal bar to arrive at the figure on the right. Compute A and then compute all x such that $Ax = 0$. Is this configuration stable?

Problem 3: (5+5+5 = 15 points)
Prove or disprove that these are subspaces of $\mathbb{R}^3$.

(a) $W_1 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 = 2a_3 \text{ and } a_2 = -7a_3\}$.

(b) $W_2 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 - 4a_2 + 5a_3 = 3\}$.

(c) $W_3 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 - 4a_2 + 5a_3 = 0\}$.

Problem 4 (5+5+10=20 points) We wish to show that $N(A) = N(A^T A)$ regardless of A.

(a) For arbitrary A show that $N(A) \subset N(A^T A)$, i.e., that if $Ax = 0$ then $A^T Ax = 0$.

(b) For arbitrary A show that $N(A^T A) \subset N(A)$, i.e., that if $A^T Ax = 0$ then $Ax = 0$.

(c) Let $K \in \mathbb{R}^{m \times m}$ be a diagonal matrix with positive diagonal entries and let $A \in \mathbb{R}^{m \times n}$. Show that $N(A) = N(A^T KA)$. (Hint: $A^T KA = \widetilde{A}^T \widetilde{A}$. What is $\widetilde{A}$?)