8/25/2021 : First Day of Class

→ intro to 335
→ vectors, matrices, operations, norms.

For our purposes, we first will think about vectors as a list of numbers:

$$\underline{v} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}.$$

The numbers do not all have to be integers:

$$\underline{w} = \begin{pmatrix} 5 \\ \pi \end{pmatrix}.$$

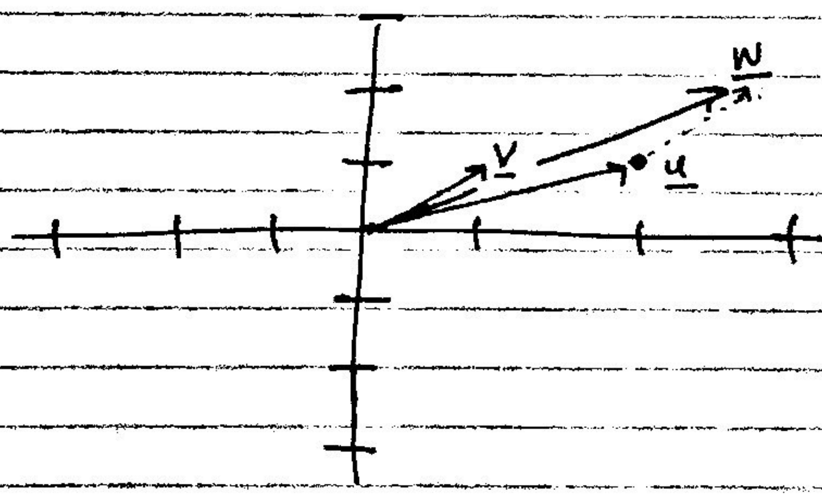
We can do "operations" with vectors, like <u>add</u> them or <u>scale</u> them.

Vector addition:

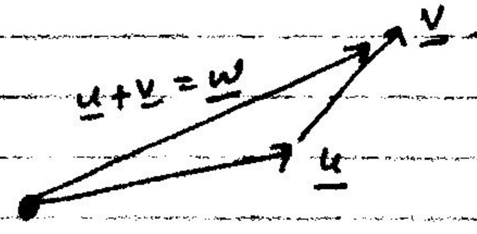
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\underline{u} + \underline{v} = \underline{w}$$

Geometric picture: plot the first number, or first component of the vector, on the x-axis, and the second component on the y axis:

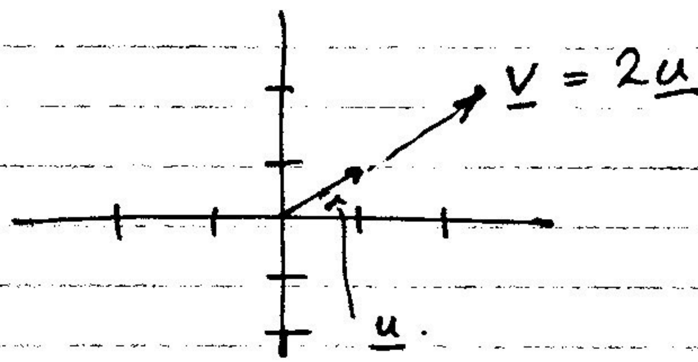


Blow up picture:



Scaling vectors (scalar multiplication):

$$2\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$$



We can collect vectors into a set, and with vector addition and scalar multiplication defined, we have a vector space:

<u>Definition</u>: The space $\mathbb{R}^n$ is the collection of vectors with n-components, each a real number:

$$\mathbb{R}^n = \left\{ \underline{v} \text{ such that } \underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}, v_i \in \mathbb{R} \right\}.$$

Remark: A vector space is really three things

definition of scalar multiplication

$$(V , + , \cdot)$$

collection of vectors

definition of vector addition

(maybe four things if you also want to include the "scalar field")

In this class, $V = \mathbb{R}^n$, and

Definition: let $\underline{v}, \underline{w} \in \mathbb{R}^n$, then

$$\underline{v} + \underline{w} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \\ v_n + w_n \end{pmatrix}.$$

Definition: let $\underline{v} \in \mathbb{R}^n$, $c \in \mathbb{R}$, then

$$c\,\underline{v} = \begin{pmatrix} c v_1 \\ c v_2 \\ \vdots \\ c v_n \end{pmatrix}.$$