Syllabus for Numerical Analysis, Spring 2019 MATH-UA 0252-003

Prerequisites

Knowledge of undergraduate linear algebra and calculus. MATLAB will be used as the primary language and you will be expected to master it (or master Python or Julia) in the first few weeks (see resources below). Before you come to class, obtain MATLAB via the NYU-wide license.

Description

We will cover classical topics in Numerical Analysis: The solution of linear and non-linear equations, conditioning, least squares, numerical computation of eigenvalues, interpolation, quadrature, and numerical methods for ODEs. The course will have a focus on the analysis of numerical methods, but also require you to use numerical software (Matlab, Python, or Julia). If you are not familiar with any of these languages, the recitation will give an introduction to MATLAB during the first weeks.

Materials

- Primary textbook: Endre Suli and David Mayers: An Introduction to Numerical Analysis. Cambridge University Press, 2003. The PDF is freely available to you via the NYU library subscription from Cambridge University Press.
- Secondary textbook for numerical linear algebra: L.N. Trefethen and D. Bau, III, *Numerical Linear Algebra*, SIAM (1997), available for purchase here. Also available as a reserve item at the Courant library.

If you need to brush up on MATLAB, you should consider these two books:

- MATLAB Guide by D.J. Higham and N.J. Higham, SIAM, 2000.
- Numerical Computing with MATLAB by Cleve Moler, available for free in PDF from MATLAB.
- An Introduction to Programming and Numerical Methods in MATLAB by Stephen R. Otto and James P. Denier, Springer, 2005. The PDF is freely available to you via the NYU library subscription at SpringerLink.

Grading

The overall course grade will be determined based on homework, quizzes, a midterm and a final exam. The following breakdown will be used:

- 30% homework
- 10% quizzes
- 25% midterm
- 35% final exam

NYU's academic integrity policies will be strictly enforced for homework assignments. While the lectures and textbook are meant to be focused on theoretical numerical analysis, computing on your own will form an essential part of the learning process and your own applied mathematics training. You are welcome to discuss problems, talk to your colleagues and consult other sources when you get stuck, as long as you explicitly acknowledge any help that you receive from any source, but you must write all of your homework solutions and every line of code yourself (including making your own figures). There is no substitute for debugging your own code — looking at or copying someone else's code is not the same.

Detailed list of topics

- 1. Solution of nonlinear scalar equations
 - (a) fixed point method, convergence speed
 - (b) Newton's method
 - (c) secant and bisection method
- 2. Solution of linear systems
 - (a) Gaussian elimination, LU factorization, forward and backward substitution, pivoting
 - (b) Norms and condition number of a matrix and error bounds
 - (c) Linear least squares and QR factorization
- 3. Eigenvalue Problems
 - (a) Power method, inverse iteration
 - (b) QR algorithm for eigenvalues
 - (c) Householder and Givens for tridiagonalization

- 4. Interpolation and Quadrature
 - (a) Lagrange and Hermite interpolation
 - (b) Convergence of interpolation
 - (c) Newton Cotes quadrature
 - (d) Orthogonal polynomials and Gauss quadrature
- 5. Initial value problems
 - (a) Existence of solutions
 - (b) Euler's method (explicit and implicit)
 - (c) Trapezoidal rule, Runge Kutta
 - (d) Multistep methods