Hyperelastic immersed boundary finite element heart model

Margaret Anne Smith, Charles Puelz, Simone Rossi, Boyce Griffith

Department of Mathematics, University of North Carolina, Chapel Hill

Motivation

Clinical intervention in heart disease may benefit from non invasive computational experimentation. This work describes a model of the full heart. Blood is modeled as an incompressible viscous fluid and the heart structures are taken to be hyperelastic solids.
Fluid/structure interaction is approximated by the immersed boundary method and results are presented for early ventricular systole.

Modeling and numerical approach

The total Cauchy–stress for the fluid/solid system is:

$$\boldsymbol{\sigma}(\boldsymbol{x},t) = \boldsymbol{\sigma}^f(\boldsymbol{x},t) + \begin{cases} \boldsymbol{\sigma}^s(\boldsymbol{x},t) & \text{if } \boldsymbol{x} \in \chi(U,t) \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

where χ is the motion map, $\boldsymbol{\sigma}^f$ is the stress for a viscous incompressible fluid, and U in the reference configuration of the solid.

The governing equations in strong form are [Boffi et al., 2008]:

$$\begin{aligned} \rho\left(\frac{\partial \boldsymbol{u}}{\partial t}(\boldsymbol{x},t) + \boldsymbol{u}(\boldsymbol{x},t)\cdot\nabla\boldsymbol{u}(\boldsymbol{x},t)\right) &= -\nabla p(\boldsymbol{x},t) + \mu\Delta\boldsymbol{u}(\boldsymbol{x},t) + \boldsymbol{f}(\boldsymbol{x},t), \\ \nabla\cdot\boldsymbol{u}(\boldsymbol{x},t) &= 0, \\ \boldsymbol{f}(\boldsymbol{x},t) &= \int_U \nabla_{\boldsymbol{X}}\cdot\boldsymbol{P}^s(\boldsymbol{X},t)\,\delta(\boldsymbol{x}-\chi(\boldsymbol{X},t))d\boldsymbol{X} \\ &\quad - \int_{\partial U} \boldsymbol{P}^s(\boldsymbol{X},t)\boldsymbol{N}(\boldsymbol{X})\,\delta(\boldsymbol{x}-\chi(\boldsymbol{X},t))dA(\boldsymbol{X}), \\ \mathcal{J}(\boldsymbol{u}) = \frac{\partial\chi}{\partial t}(\boldsymbol{X},t) &= \int_U \boldsymbol{u}(\boldsymbol{x},t)\,\delta(\boldsymbol{x}-\chi(\boldsymbol{X},t))d\boldsymbol{X}. \end{aligned}$$

$\boldsymbol{u}$ = Eulerian velocity field
$\boldsymbol{p}$ = pressure
$\boldsymbol{F} = \partial\chi/\partial\boldsymbol{X}$ = deformation gradient
$\boldsymbol{P}^s = J\boldsymbol{\sigma}^s\boldsymbol{F}^{-T}$ = first Piola–Kirchoff stress

The strain–energy functional for the **heart myocardium** is taken as:

$$\begin{aligned} W &= \tfrac{1}{2}c\left(\exp Q - 1\right), \\ Q &= b_f\tilde{E}_{11}^2 + b_t(\tilde{E}_{22}^2+\tilde{E}_{33}^2+\tilde{E}_{23}^2+\tilde{E}_{32}^2) + b_{fs}(\tilde{E}_{12}^2+\tilde{E}_{21}^2+\tilde{E}_{13}^2+\tilde{E}_{31}^2). \end{aligned}$$

with $\tilde{E}_{ij}$ components of the Green–Lagrange strain tensor $\frac{1}{2}(\boldsymbol{F}^T\boldsymbol{F}-\boldsymbol{I})$ rotated so the first unit vector aligns with a specified fiber direction $\boldsymbol{f}_0$ [Guccione et al., 1995].

Valves, chordae, and papillary muscles models use a neohookean constitutive law $\boldsymbol{P}^s = \mu_e(\boldsymbol{F}-\boldsymbol{F}^{-T})$.

A finite volume type (MAC) discretization is used for the fluid.

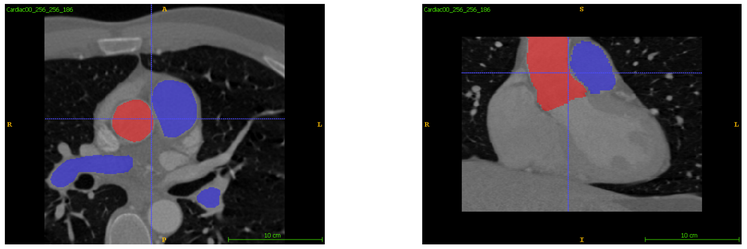
The total elastic force density is approximated using finite elements:

$$\int_U \boldsymbol{G}(\boldsymbol{X},t)\cdot\boldsymbol{V}_h(\boldsymbol{X})d\boldsymbol{X} = -\int_U \boldsymbol{P}^s(\boldsymbol{X},t)\cdot\nabla_{\boldsymbol{X}}\boldsymbol{V}_h(\boldsymbol{X})d\boldsymbol{X}, \quad \text{and } \boldsymbol{g} = \mathcal{S}_h(\boldsymbol{G}),$$

where $\boldsymbol{g}(\boldsymbol{x},t)$ is a fluid forcing term. The adjoint of $\mathcal{S}_h$ restricts the velocity to the finite element mesh, i.e. $\mathcal{S}_h^* \approx \mathcal{J}$ [Griffith et al., 2017].

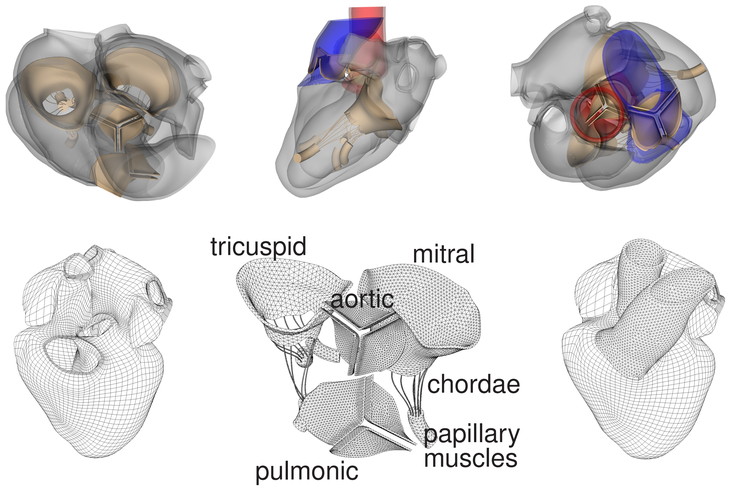
Construction

Manual segmentation of ascending aorta (red) and pulmonary artery (blue) from computed tomography (CT) dataset.



Axial view (left) and coronal view (right).

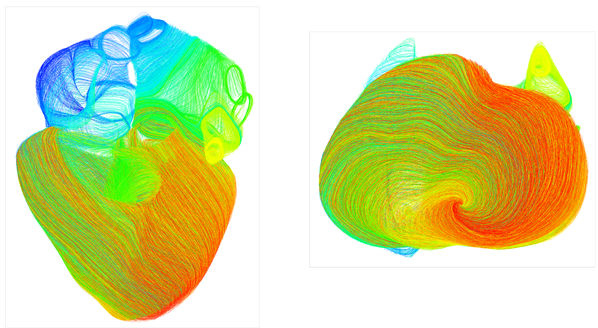
3D heart model geometry and meshes with idealized valves configured as in ventricular systole with atrioventricular valves open and both aortic and pulmonic valves closed.



The fiber vector field $\boldsymbol{f}_0$ is visualized by solving the following ODE for a set of uniformly spaced seed points:

$$\alpha'(t) = \boldsymbol{f}_0(\alpha(t)), \quad \alpha(0) = \text{seed point}.$$

Colors correspond to different seed points.

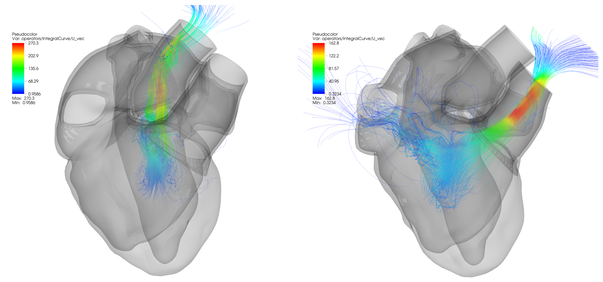


Acknowledgements: We thank Paul Segars from Duke for providing the heart mesh and CT data.

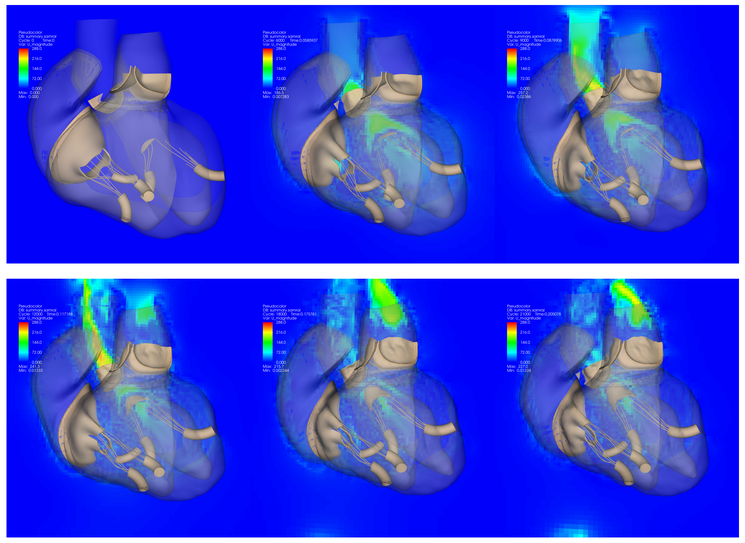
Funding: NIH R01HL117063, NSF OAC 1652541: CAREER, NSF OAC 1450327.

Results

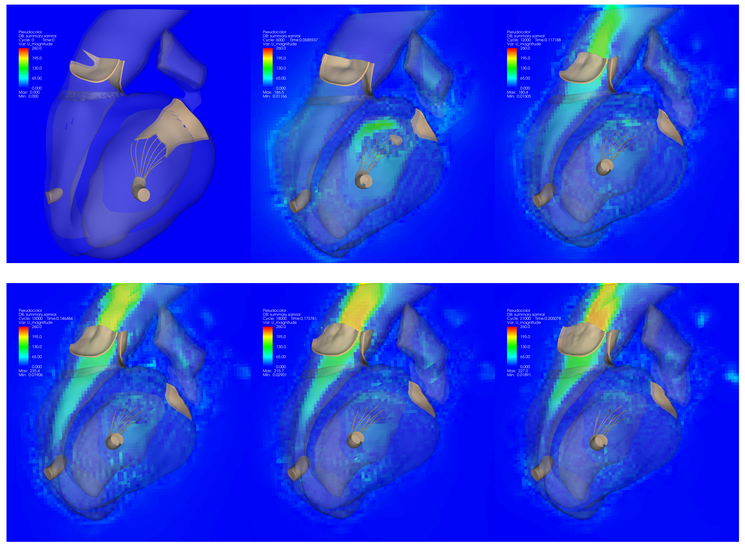
Velocity streamline plots with seed points originating in the left and right ventricles. Color indicates velocity magnitude.



Fluid velocity in three-dimensional heart model sliced through the ascending aorta.



Fluid velocity in three-dimensional heart model sliced through the ascending pulmonary artery.



Contact: cpuelz@unc.edu